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CALCULATION OF EFFECTIVE THERMAL RADIATION ABSORPTION COEFFICIENT OF A CAVITY WITH DIFFUSELY REFLECTING WALLS

S. B. Kiselev

UDC 536.3

The system of integral equations of radiation heat exchange in a closed cavity is solved numerically.

One of the basic requirements of a calorimeter for radiant heat fluxes is the total absorption of all radiation incident on its entrance opening, independently of the spectral composition and direction of the radiation. The most effective method of increasing the absorption of radiation is the use of cavities of different configurations to collect the radiation. The geometry of a cavity can be changed so as to make its radiation characteristics approach those of a black body as closely as possible. The actual characteristics of the cavity can be determined either experimentally or theoretically, but the experimental arrangements for determining the absorptance of a cavity are so complex that only the theoretical solution of this problem is practical.

The effective thermal radiation absorption coefficient of a cavity of any configuration is defined as the ratio

$$\varepsilon_{\text{eff}} = 1 - \frac{Q_{\text{ref}}}{Q_{\text{in}}}, \tag{1}$$

where

$$Q_{\text{ref}} = \sum_{i=1}^{N} \int_{A_i} \int_{A_0} (\varphi_i(\mathbf{r}_i) - f_i(\mathbf{r}_i)) K_{i0} dA_i dA_0 \tag{2}$$

is the reflected heat, and

$$Q_{\text{in}} = \int_{A_0} \varphi_0(\mathbf{r}_0)\, dA_0 \tag{3}$$

is the incident heat. Here $\varphi_i(\mathbf{r}_i)$ is an unknown function characterizing the flux density of effective radiation from the i-th zone of the cavity surface (the subscript o refers to the opening); $f_i(\mathbf{r}_i)$ is a known function which characterizes the self-radiation of the cavity surface.

In order to find the unknown function $\varphi_i(\mathbf{r}_i)$, and consequently to determine the radiation characteristics of the cavity, it is necessary to solve the radiation heat-exchange problem in a closed cavity. By using the generalized zonal method this problem is reduced to the solution of a system of integral equations of the form [1]

$$\varphi_i(\mathbf{r}_i) = g_i(\mathbf{r}_i) + \lambda_i \sum_{j=1}^{N} \int_{A_j} \varphi_j(\mathbf{r}_j) K_{ij} dA_j \quad (i = 1, 2, \ldots, N), \tag{4}$$

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where $\lambda_i = 1 - \varepsilon_i$ is the reflection coefficient of the i-th zone of the cavity surface; $g_i(\mathbf{r}_i)$, distribution of thermal flux density or temperature over the i-th zone (specified in the boundary conditions); K_{ij}, kernel of the integral equation, and is related to the elementary diffuse angular coefficient by the expression $dF_{dA_i - dA_j} \equiv K_{ij} dA_j$. For the geometry of the system shown in Fig. 1 the quantity $dF_{dA_i - dA_j}$ is given by the expression

$$dF_{dA_2 - dA_1} = \frac{(\mathbf{n}_1 \mathbf{r}_{12})(\mathbf{n}_2 \mathbf{r}_{12})}{\pi \mathbf{r}^4} dA_1.$$

As shown in [2], solving Eq. (4) is equivalent to finding the extremum of the functional

$$I = \sum_{j=1}^{N} \iint_{A_j A_j} \varphi_j^2 K_{jj} dA_j dA_j + 2 \sum_{j=2}^{N} \Big(\sum_{i=1}^{j-1} \iint_{A_i A_j} \varphi_i \varphi_j K_{ij} dA_i dA_j \Big) - \sum_{j=1}^{N} \frac{1}{\lambda_j} \int_{A_j} \varphi_j^2 dA_j + 2 \sum_{j=1}^{N} \int_{A_j} g_j \varphi_j dA_j. \tag{5}$$

If the functions $\varphi_1, \ldots, \varphi_N$ are determined so as to make the functional (5) an extremum, they are also the solution of the system of integral equations (4). Since it is difficult to find the exact solutions for the functions $\varphi_1(\mathbf{r}_1), \ldots, \varphi_N(\mathbf{r}_N)$, we use the Ritz approximation method [3] in which each of the functions $\varphi_i(\mathbf{r}_i)$ is represented as a linear combination of M appropriately chosen functions $\psi_{im}(\mathbf{r}_i)$:

$$\varphi_i(\mathbf{r}_i) = \sum_{m=1}^{N} c_{im} \psi_{im}(\mathbf{r}_i) \quad (i = 1, 2, \ldots, N), \tag{6}$$

where the constants c_{im} ($i = 1, 2, \ldots, N$; $m = 1, 2, \ldots, M$) are found from the condition

$$\frac{\partial I}{\partial c_{im}} = 0 \quad (i = 1, 2, \ldots, N, \ m = 1, 2, \ldots, M). \tag{7}$$

This system contains MN algebraic equations with MN unknown coefficients. The accuracy of the solution obtained can be increased by increasing the number of terms in expansion (6).

The method described is used to calculate the effective absorption coefficient of the cylindrically symmetric composite cavity shown in Fig. 2. Equations (4) are solved subject to the following assumptions and boundary condition:

1) the quantities $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$ do not depend on the wavelength or direction of the incident radiation;

2) radiation is reflected and emitted diffusely by the cavity walls;

3) $f_i(\mathbf{r}_i) = 0$, the self-radiation of the cavity walls is eliminated from the solution;

4) the opening is replaced by an ideal black surface with a uniformly distributed effective radiation flux density;

$$\varphi_0(\mathbf{r}_0) = 1 \quad (\lambda_0 = 0, \ \varphi_0(\mathbf{r}_0) \equiv g_0(\mathbf{r}_0));$$

5) the basis functions in expansion (6) are chosen in the form

$$\psi_{im}(\mathbf{r}_i) = l_i^{m-1}, \tag{8}$$

where $l_1 = r_1$, $l_2 = r_2$, $l_3 = r_3$. The subscripts correspond to the numbers of the surface shown in Fig. 2.

Using the assumptions and boundary conditions, we obtain from (5)-(7)

$$AX = B, \tag{9}$$

where X is a column vector of the unknown coefficients

$$x_k = c_{jn}, \ k = (j-1) \times M + n \quad (j = 1, 2, 3; \ n = 1, 2, \ldots, M); \tag{10}$$

A is the matrix of the coefficients with elements

$$a_{lk} = 2 \iint_{A_i A_j} (l_i)^{m-1} (l_j)^{n-1} K_{ij} dA_i dA_j \quad \text{for} \quad i \neq j, \tag{11}$$

$$a_{lk} = \iint_{A_i A_i} [(l_i)^{m-1} (l_i')^{n-1} + (l_i')^{m-1} (l_i)^{n-1}] K_{ii} dA_i dA_i' - \frac{2}{1-\varepsilon} \int_{A_i} (l_i)^{m+n-2} dA_i \quad \text{for} \quad i = j; \tag{12}$$

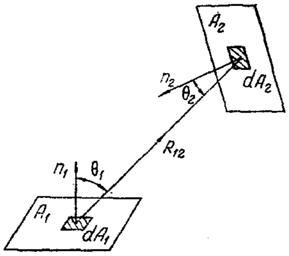

Fig. 1

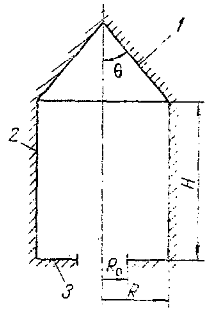

Fig. 2

Fig. 1. Coordinates for defining the diffuse angular coefficient.

Fig. 2. Geometric characteristics of cavity: 1-3) surface numbers.

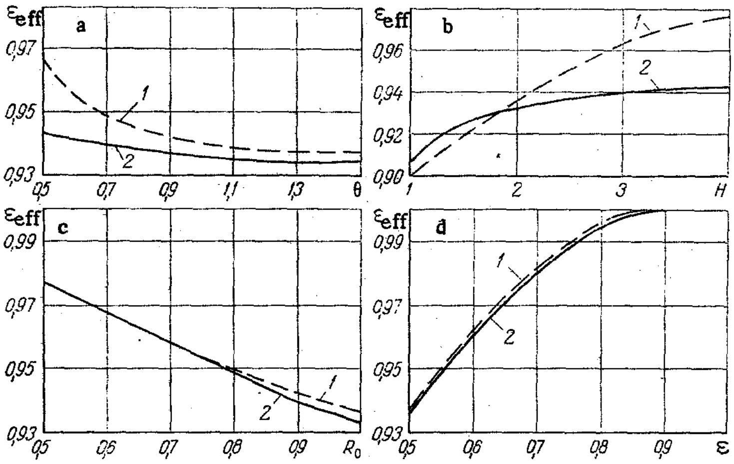

Fig. 3. Effective absorption coefficient of a cavity as a function of: a) angle of taper; b) height; c) radius of opening; d) emissivity of walls: 1) parallel radiation; 2) diffuse radiation.

B is a column vector with the following elements:

$$b_e = -2(1-\varepsilon)\int\limits_{A_i}\int\limits_{A_0}(l_i)^{m-1}K_{i0}dA_i dA_0,\quad e=(i-1)\times M+m \qquad (13)$$

$$(i=1,\ 2,\ 3,\ m=1,\ 2,\ \ldots,\ M).$$

The system of linear algebraic equations (9) was solved numerically by computer. All the surface integrals appearing in Eqs. (2), (11), (12), and (13) were calculated by successive applications of Gauss's formula [4] with 7, 6, 5, and 4 nodal points, respectively. With this integration scheme Q_{ref} was evaluated with an error of no more than 20% for M = 3, which corresponds to a relative error $\Delta\varepsilon_{eff} \leq 2\%$, for $\varepsilon_{eff} \geq 0.9$. A further increase in the number of terms in expansion (6) does not lead to a significant increase in accuracy. The accuracy of the calculation can be increased to any specified value by using Gauss's formula with a larger number of nodal points, or by employing other more accurate integration schemes. It should be noted, however, that this leads to a sharp increase in computation time. For example, doubling the number of nodal points leads to a 16-fold increase in machine time. On the other hand, with an increase in ε_{eff} the accuracy of its calculation is sharply increased as the result of the small role played by Q_{ref} in the overall heat balance [cf. Eq. (1)], and therefore the accuracy achieved is quite sufficient for practical purposes. The calculations were performed for two limiting cases: 1) the opening of the cavity radiates diffusely; 2) the radiation enters the cavity parallel to the axis of symmetry. In all the calculations the cavity parameters were as follows: angle of taper $\theta = 1.571$ rad; height H = 2.0, radius R = 1.0 (scale of linear dimensions); radius of opening $R_0 = 1.0$; emissivity of walls $\varepsilon = 0.5$.

The results of the calculations for cavities of various configurations are shown in Fig. 3. It is clear from the figures that the effective absorption coefficient of the cavity is higher for parallel than for diffuse radiation, except for a cylindrical cavity with $H < 2.0$. With increasing depth of the cavity, or a decrease in the angle of taper, ε_{eff} approaches a certain limiting value asymptotically ($\varepsilon_{eff} = 1$ for parallel radiation, and $\varepsilon_{eff} = 0.943$ for diffuse radiation). Therefore, increasing H beyond 4.0 or decreasing θ below 0.5 for diffuse radiation increases ε_{eff} only slightly. The value of ε_{eff} is more effectively increased by increasing the emissivity of the cavity walls and decreasing the radius of the cavity opening. For parallel radiation decreasing the angle of taper θ below 0.5 is also effective in increasing ε_{eff}. By choosing optimum values of all four parameters it is possible to produce a calorimeter for thermal radiation with characteristics closely approaching those of a black body.

NOTATION

θ, angle of taper of cavity; H, height of cavity; R, radius; R_0, radius of opening of cavity; ε, emissivity of cavity walls; ε_{eff}, effective emissivity of cavity; Q_{in}, incident heat; Q_{ref}, reflected heat; λ, reflection coefficient of cavity walls.

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SOME FEATURES OF THE THERMALLY CONCENTRATED CONVECTIVE MOTION OF A HARDENING BINARY MELT AND THE IMPURITY DISTRIBUTION

P. F. Zavgorodnii

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Some features of the thermally concentrated convective motion of a binary melt, hardening in a closed rectangular region with movable boundaries, and the impurity distribution are investigated numerically.

It was shown in [1] that the impurity distribution in the hardening part of a crystallizing fixed melt is mainly determined by the nature of the change in the impurity concentration at the boundary between the hard and liquid phases. It was established in [2] that convective mixing of the liquid nucleus due to its temperature nonuniformity has a considerable effect on the nature of the impurity distribution at the phase-transition boundary and, consequently, on the impurity distribution in the hardening part of the crystallizing melt.

However, some features of the hardening of a binary melt were ignored in [1, 2]. Thus, when a binary melt hardens a concentrational nonuniformity develops in the liquid nucleus together with a temperature nonuniformity, due to the difference in the solubility of the impurity in the solid and liquid phases. The result of the combined action of the temperature and concentration nonuniformities will be the occurrence and development of a thermally concentrated gravitational convective motion in the liquid nucleus of the hardening alloy, the features of which should also manifest themselves in the nature of the impurity distribution.

Consider a rectangular region filled with melt with initial temperature $T_0 > T_K$ and an initial impurity content c_0, with relative dimensions $l_1 = L_1/x_0$, $l_2 = L_2/x_0$. The region in which the melt exists is situated in space such that $0 \leq x_1 \leq L_1$, $0 \leq x_2 \leq L_2$, and the direction of the acceleration due to gravity determines the positive di-

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